جـامـعـة بنهــــا

كلية الحاسبات والذكاء الإصطناعي

Probability and Statistics

Lecture 06

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Chapter 2: Random Variable

- Discrete Random Variables.
- Probability Mass Functions.
- Cumulative Distribution Functions.
- Discrete R.V. (Mean and Variance).
- Continuous Random Variables.
- Probability Density Functions.
- Continuous R.V. (Mean and Variance).
- Joint Probability Distributions.

Joint Probability Mass Function:

If X and Y are two discrete random variables, the joint probability mass function is denoted as $f_{XY}(x, y)$, satisfies

1) $f_{XY}(x, y) \geq 0$

2) $\sum_X \sum_Y f_{XY}(x, y) = 1$

3) $f_{XY}(x, y) = P(X = x, Y = y)$

Joint Prob. Mass Fun. (2/11)

Review Example 1

$$S = \{HH, HT, TH, TT\}$$

X	2	1	1	0	**number of heads**
Y	0	1	1	2	**number of tails**

Joint Prob. Mass Fun. (3/11)

Review Example 1

$S = \{HH, HT, TH, TT\}$

X	2	1	1	0
Y	0	1	1	2

number of heads

number of tails

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0

joint probability mass function $f_{XY}(x, y)$

Joint Prob. Mass Fun. (4/11)

Marginal Probability Distributions

The marginal distributions of the random variable X alone is:

$$f_X(x) = \sum_y f_{XY}(x, y)$$

The marginal distributions of the random variable Y alone is:

$$f_Y(y) = \sum_x f_{XY}(x, y)$$

Joint Prob. Mass Fun. (5/11)

Review Example2

y \ x	**0**	**1**	**2**
0	0	0	1/4
1	0	2/4	0
2	1/4	0	0
$f_X(x)$	**1/4**	**2/4**	**1/4**

Find:

1) $f_X(x)$

Joint Prob. Mass Fun. (5/11)

Review Example2

$f_X(x)$

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

Joint Prob. Mass Fun. (5/11)

Review Example3

y \ x	**0**	**1**	**2**	$f_Y(y)$
0	0	0	1/4	**1/4**
1	0	2/4	0	**2/4**
2	1/4	0	0	**1/4**

Find:

2) $f_Y(y)$

Joint Prob. Mass Fun. (5/11)

Review Example3

$f_Y(y)$

y	0	1	2
$f_Y(y)$	$1/4$	$2/4$	$1/4$

Mean from a Joint Distribution (Discrete):

Mean for the random variable X alone is:

$$E(X) = \sum_{x} x f_X(x)$$

Mean for the random variable Y alone is:

$$E(Y) = \sum_{y} y f_Y(y)$$

Variance from a Joint Distribution (Discrete):

Variance for the random variable X alone is:

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

and s.d. of $X = \sigma_X = \sqrt{V(X)}$

Variance for the random variable Y alone is:

$$V(Y) = E(Y^2) - \left(E(Y)\right)^2$$

and s.d. of $Y = \sigma_Y = \sqrt{V(Y)}$

Joint Prob. Mass Fun. (8/11)

Example1 (1/4): (from the previous example)

$f_X(x)$

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

Find:

$E(X)$

$V(X)$

σ_X

Joint Prob. Mass Fun. (8/11)

Example1 (2/4): (from the previous example)

$f_X(x)$

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

1:

$$E(X) = \sum_x x f_X(x) = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{2}{4}\right) + (2)\left(\frac{1}{4}\right) = 1$$

Joint Prob. Mass Fun. (8/11)

Example1 (3/4): (from the previous example)

$f_X(x)$

x	**0**	**1**	**2**
$f_X(x)$	**1/4**	**2/4**	**1/4**

2:

$$V(X) = E(X^2) - \left(E(X)\right)^2$$

$$E(X^2) = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{2}{4}\right) + (4)\left(\frac{1}{4}\right) = \frac{6}{4} = 1.5$$

Joint Prob. Mass Fun. (8/11)

Example1 (3/4): (from the previous example)

$f_X(x)$

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

2:

$$E(X^2) = (0)\left(\frac{1}{4}\right) + (1)\left(\frac{2}{4}\right) + (4)\left(\frac{1}{4}\right) = \frac{6}{4} = 1.5$$

$$V(X) = 1.5 - (1)^2 = 0.5$$

Joint Prob. Mass Fun. (8/11)

Example1 (4/4): (from the previous example)

$f_X(x)$

x	0	1	2
$f_X(x)$	1/4	2/4	1/4

3:

$$V(X) = 1.5 - (1)^2 = 0.5$$

$$\sigma_X = \sqrt{0.5} = 0.7071$$

Conditional Probability Distributions (Discrete):

The random variables X and Y are discrete, the conditional probability mass function of X given $Y = y$ is:

$$f_{X|y}(x) = \frac{f_{XY}(x, y)}{f_Y(y)}$$

for $f_Y(y) > 0$

Joint Prob. Mass Fun. (9/11)

Conditional Probability Distributions (Discrete):

The random variables X and Y are discrete, the conditional probability mass function of X given $Y = y$ is:

$$f_{X|y}(x) = \frac{f_{XY}(x, y)}{f_Y(y)}$$

1

$$\boldsymbol{f_{X|2}(x) = f_{X|Y=2}(x) = f_{XY}(x, 2)/f_Y(2)}$$

The conditional probability distribution of X given that $Y = 2$.

Joint Prob. Mass Fun. (9/11)

Conditional Probability Distributions (Discrete):

The random variables X and Y are discrete, the conditional probability mass function of X given $Y = y$ is:

$$f_{X|y}(x) = \frac{f_{XY}(x, y)}{f_Y(y)}$$

2

$$\boldsymbol{f_{Y|3}(y) = f_{Y|X=3}(y) = f_{XY}(3, y)/f_X(3)}$$

The conditional probability distribution of Y given that $X = 3$.

Joint Prob. Mass Fun. (10/11)

Example2:

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Find:

$f_X(x)$

$E(Y)$

σ_X

$f_{X|3}(x)$

$f_{Y|1.5}(y)$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (1/17):

$f_{XY}(x, y)$

1:

$f_X(x) \rightarrow \text{marginal } X$

y \ x	**1**	**1.5**	**2.5**	**3**
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**

Example2 – Answer (1/17):

x	1	1.5	2.5	3
$f_X(x)$	$1/4$	$3/8$	$1/4$	$1/8$

1:

$f_X(x)$ $\rightarrow$ marginal X

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (2/17):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

2:

$E(Y) \rightarrow \text{Mean } Y$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (2/17):

$f_{XY}(x,y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

2:

$$E(Y)=\sum_{y} y f_Y(y)$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (3/17):

$f_{XY}(x, y)$

2:

$$E(Y) = \sum_{y} y f_Y(y)$$

y \ x	1	1.5	2.5	3	$f_Y(y)$
1	$1/4$	0	0	0	**$1/4$**
2	0	$1/8$	0	0	**$1/8$**
3	0	$1/4$	0	0	**$1/4$**
4	0	0	$1/4$	0	**$1/4$**
5	0	0	0	$1/8$	**$1/8$**

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (3/17):

2:

$$E(Y) = \sum_y y f_Y(y)$$

y	$f_Y(y)$
1	1/4
2	1/8
3	1/4
4	1/4
5	1/8

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (4/17):

y	$f_Y(y)$
1	1/4
2	1/8
3	1/4
4	1/4
5	1/8

2:

$$E(Y) = \sum_{y} y f_Y(y)$$

$$E(Y) = (1)\left(\frac{1}{4}\right) + (2)\left(\frac{1}{8}\right) + (3)\left(\frac{1}{4}\right) + (4)\left(\frac{1}{4}\right) + (5)\left(\frac{1}{8}\right) = \frac{23}{8}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (4/17):

y	$f_Y(y)$
1	1/4
2	1/8
3	1/4
4	1/4
5	1/8

2:

$$E(Y) = \sum_y y f_Y(y)$$

$$E(Y) = \frac{23}{8} = 2.875$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (5/17):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

<u>3:</u>

$\sigma_X \rightarrow$ s.d. of X

$= \sqrt{V(X)}$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (5/17):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

<u>3:</u>

$\sigma_X \rightarrow$ s.d. of X

$= \sqrt{V(X)}$

$= E(X^2) - (E(X))^2$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (6/17):

Recall

1:

$f_X(x) \rightarrow$ marginal X

x	1	1.5	2.5	3
$f_X(x)$	1/4	3/8	1/4	1/8

3:

$\sigma_X \rightarrow$ s.d. of X

$$= \sqrt{V(X)}$$

Example2 – Answer (7/17):

Recall

1:

$f_X(x) \rightarrow$ marginal X

x	1	1.5	2.5	3
$f_X(x)$	1/4	3/8	1/4	1/8

3:

$\sigma_X \rightarrow$ s.d. of X

$$= \sqrt{V(X)}$$

$$E(X) = \sum_x x f_X(x)$$

$$= (1)\left(\frac{1}{4}\right) + (1.5)\left(\frac{3}{8}\right) + (2.5)\left(\frac{1}{4}\right) + (3)\left(\frac{1}{8}\right) = 1.8125$$

Example2 – Answer (8/17):

Recall

1:

$f_X(x) \rightarrow \text{marginal } X$

x	1	1.5	2.5	3
$f_X(x)$	1/4	3/8	1/4	1/8

3:

$\sigma_X \rightarrow \text{s.d. of } X$

$= \sqrt{V(X)}$

$$E(X^2) = \sum_x x^2 f_X(x)$$

$$= (1)\left(\frac{1}{4}\right) + (2.25)\left(\frac{3}{8}\right) + (6.25)\left(\frac{1}{4}\right) + (9)\left(\frac{1}{8}\right)$$

$$= 3.78125$$

Example2 – Answer (9/17):

$E(X) = 1.8125$

$E(X^2) = 3.78125$

3:

$\sigma_X \rightarrow$ s.d. of X

$$V(X) = 3.78125 - (1.8125)^2 = 0.49609375$$

$$\sigma_X = \sqrt{0.49609375} = 0.704339$$

Example2 – Answer (10/17):

4:

$f_{X|3}(x)$

$The\ conditional\ probability\ distribution\ of\ X\ given\ that\ Y = 3.$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (11/17):

$f_{XY}(x,y)$

y \ x	1	1.5	2.5	3
1	$1/4$	0	0	0
2	0	$1/8$	0	0
3	0	$1/4$	0	0
4	0	0	$1/4$	0
5	0	0	0	$1/8$

<u>**4:**</u>

$f_{X|3}(x)$

$$f_{X|y}(x) = \frac{f_{XY}(x,y)}{f_Y(y)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (11/17):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

4:

$f_{X|3}(x)$

$y = 3$

$$f_{X|y}(x) = \frac{f_{XY}(x, y)}{f_Y(y)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (11/17):

y \ x	1	1.5	2.5	3
1	$1/4$	0	0	0
2	0	$1/8$	0	0
3	0	$1/4$	0	0
4	0	0	$1/4$	0
5	0	0	0	$1/8$

$f_{XY}(x,y)$

4:

$f_{X|3}(x)$

$$f_{X|3}(x) = \frac{f_{XY}(x,3)}{f_Y(3)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (12/17):

$f_{XY}(x, y)$

4:

$f_{X|3}(x)$

$$f_{X|3}(x) = \frac{f_{XY}(x,3)}{f_Y(3)}$$

y \ x	1	1.5	2.5	3	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (12/17):

$f_{XY}(x, y)$

y \ x	**1**	**1.5**	**2.5**	**3**	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**

4:

$f_{X|3}(x)$

$y = 3$

$$f_{X|3}(x) = \frac{f_{XY}(x, 3)}{f_Y(3)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (13/17):

$f_{XY}(x, y)$

4:

$f_{X|3}(x)$

$y = 3$

$$f_{X|3}(1) = \frac{f_{XY}(1,3)}{f_Y(3)} = \frac{0}{1/4}$$

y \ x	**1**	**1.5**	**2.5**	**3**	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**
$f_{X\|3}(x)$	**0**				

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (13/17):

$f_{XY}(x,y)$

4:

$f_{X|3}(x)$

$y = 3$

$y \backslash x$	**1**	**1.5**	**2.5**	**3**	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**
$f_{X\|3}(x)$	**0**	**1**			

$$f_{X|3}(1.5) = \frac{f_{XY}(1.5,3)}{f_Y(3)} = \frac{1/4}{1/4}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (13/17):

$f_{XY}(x,y)$

4:

$f_{X|3}(x)$

$y = 3$

$y \backslash x$	**1**	**1.5**	**2.5**	**3**	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**
$f_{X\|3}(x)$	**0**	**1**	**0**		

$$f_{X|3}(2.5) = \frac{f_{XY}(2.5,3)}{f_Y(3)} = \frac{0}{1/4}$$

Example2 – Answer (13/17):

$f_{XY}(x,y)$

4:

$f_{X|3}(x)$

$y = 3$

$$f_{X|3}(3) = \frac{f_{XY}(3,3)}{f_Y(3)} = \frac{0}{1/4}$$

y \ x	**1**	**1.5**	**2.5**	**3**	$f_Y(y)$
1	1/4	0	0	0	**1/4**
2	0	1/8	0	0	**1/8**
3	0	1/4	0	0	**1/4**
4	0	0	1/4	0	**1/4**
5	0	0	0	1/8	**1/8**
$f_{X\|3}(x)$	**0**	**1**	**0**	**0**	

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (13/17):

$f_{XY}(x, y)$

4:

$f_{X|3}(x)$

x	1	1.5	2.5	3
$f_{X\|3}(x)$	0	1	0	0

$$f_{X|3}(x) = \frac{f_{XY}(x, 3)}{f_Y(3)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (14/17):

5:

$f_{Y|1.5}(y)$

$The\ conditional\ probability\ distribution\ of\ Y\ given\ that\ X = 1.5.$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (15/17):

y \ x	**1**	**1.5**	**2.5**	**3**
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

$f_{XY}(x, y)$

<u>5:</u>

$f_{Y|1.5}(y)$

$$f_{Y|x}(y) = \frac{f_{XY}(x, y)}{f_X(x)}$$

Example2 – Answer (15/17):

$f_{XY}(x, y)$

$y \backslash x$	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

5:

$f_{Y|1.5}(y)$

$x = 1.5$

$$f_{Y|x}(y) = \frac{f_{XY}(x, y)}{f_X(x)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (15/17):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

5:

$f_{Y|1.5}(y)$

$$f_{Y|1.5}(y) = \frac{f_{XY}(1.5, y)}{f_X(1.5)}$$

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (16/17):

$x = 1.5$

$f_{XY}(x, y)$

5:

$f_{Y|1.5}(y)$

$$f_{Y|1.5}(y) = \frac{f_{XY}(1.5, y)}{f_X(1.5)}$$

y \ x	**1**	**1.5**	**2.5**	**3**
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (17/17):

$x = 1.5$

$f_{XY}(x, y)$

5:

$f_{Y|1.5}(y)$

$$f_{Y|1.5}(y) = \frac{f_{XY}(1.5, y)}{f_X(1.5)}$$

y \ x	**1**	**1.5**	**2.5**	**3**	$f_{Y\|1.5}(y)$
1	1/4	0	0	0	**0**
2	0	1/8	0	0	**1/3**
3	0	1/4	0	0	**2/3**
4	0	0	1/4	0	**0**
5	0	0	0	1/8	**0**
$f_X(x)$	**1/4**	**3/8**	**1/4**	**1/8**	

Joint Prob. Mass Fun. (10/11)

Example2 – Answer (17/17):

5:

$f_{Y|1.5}(y)$

$$f_{Y|1.5}(y) = \frac{f_{XY}(1.5, y)}{f_X(1.5)}$$

y	$f_{Y\|1.5}(y)$
1	0
2	1/3
3	2/3
4	0
5	0

Joint Prob. Mass Fun. (11/11)

Conditional Mean and Variance (Discrete):

The conditional mean of X given $Y = y$:

$$E(X \mid y) = \sum_x x f_{X|y}(x)$$

The conditional variance of X given $Y = y$:

$$V(X \mid y) = \sum_x x^2 f_{X|y}(x) - \left(\sum_x x f_{X|y}(x)\right)^2$$

Joint Prob. Mass Fun. (11/11)

Example3:

$f_X(x)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Find:

$E(X \mid 3)$

$V(X \mid 3)$

Joint Prob. Mass Fun. (11/11)

Example3 – Answer (1/7):

$f_{XY}(x, y)$

1:

$E(X \mid 3)$

$$E(X \mid y) = \sum_{x} x f_{X|y}(x)$$

y \ x	**1**	**1.5**	**2.5**	**3**
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

Joint Prob. Mass Fun. (11/11)

Example3 – Answer (1/7):

$f_{XY}(x, y)$

$y \backslash x$	**1**	**1.5**	**2.5**	**3**
1	$1/4$	0	0	0
2	0	$1/8$	0	0
3	0	$1/4$	0	0
4	0	0	$1/4$	0
5	0	0	0	$1/8$

1:

$E(X \mid 3)$

$y = 3$

$$E(X \mid 3) = \sum_{x} x f_{X|3}(x)$$

Example3 – Answer (2/7):

Recall

1:

$E(X \mid 3)$

$y = 3$

$$E(X \mid 3) = \sum_{x} x f_{X|3}(x)$$

4:

$f_{X|3}(x)$

x	1	1.5	2.5	3
$f_{X\|3}(x)$	0	1	0	0

Example3 – Answer (3/7):

Recall

1:

$E(X \mid 3)$

4:

$f_{X|3}(x)$

x	1	1.5	2.5	3
$f_{X\|3}(x)$	0	1	0	0

$$E(X \mid 3) = \sum_x x f_{X|3}(x)$$

$$= (1)(0) + (1.5)(1) + (2.5)(0) + (3)(0) = 1.5$$

Joint Prob. Mass Fun. (11/11)

Example3 – Answer (4/7):

$f_{XY}(x, y)$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

2:

$V(X \mid 3)$

$$V(X \mid y) = \sum_x x^2 f_{X|y}(x) - \left(\sum_x x f_{X|y}(x)\right)^2$$

Joint Prob. Mass Fun. (11/11)

Example3 – Answer (4/7):

$f_{XY}(x,y)$

2:

$V(X \mid 3)$

$y = 3$

y \ x	1	1.5	2.5	3
1	1/4	0	0	0
2	0	1/8	0	0
3	0	1/4	0	0
4	0	0	1/4	0
5	0	0	0	1/8

$$V(X \mid 3) = \sum_{x} x^2 f_{X|3}(x) - \left(\sum_{x} x f_{X|3}(x)\right)^2$$

Example3 – Answer (5/7):

Recall

$$E(X \mid 3) = \sum_{x} x f_{X|3}(x) = 1.5$$

2:

$V(X \mid 3)$

$y = 3$

$$V(X \mid 3) = \sum_{x} x^2 f_{X|3}(x) - (1.5)^2$$

Example3 – Answer (6/7):

Recall

4:

$f_{X|3}(x)$

x	1	1.5	2.5	3
$f_{X\|3}(x)$	0	1	0	0

2:

$V(X \mid 3)$

$$\sum_x x^2 f_{X|3}(x) = (1)(0) + (2.25)(1) + (6.25)(0) + (9)(0) = 2.25$$

$$V(X \mid 3) = \sum_x x^2 f_{X|3}(x) - (1.5)^2$$

Joint Prob. Mass Fun. (11/11)

Example3 – Answer (7/7):

2:

$V(X \mid 3)$

$$\sum_x x^2 f_{X|3}(x) = (1)(0) + (2.25)(1) + (6.25)(0) + (9)(0) = 2.25$$

$$V(X \mid 3) = 2.25 - (1.5)^2 = 0$$

Video Lectures

All Lectures: https://www.youtube.com/playlist?list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-

Lecture #6: https://www.youtube.com/watch?v=vZLUoC_03xs&list=PLxIvc-MGOs6gW9SgkmoxE5w9vQkID1_r-&index=8

Thank You

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